# PHOTOMETRY AND ORBIT DETERMINATION

Asteroid rotation provides essential information and clues that may have led to the collisional history, their dynamic evolution resulting in their current shape and size. One of the most popular and used techniques to investigate these properties is light curve photometry. The thesis shall heavily implement this method to obtain the rotational period alongside initial orbit determination for the characteristics of these NEAs. Chapter 3 focuses more on the methodology and concept that will drive the project forward. A review of the light curve is important to provide and govern concepts presented and applied in the later chapters.

## Photometry

Photometry is applied in astronomy to measure the flux and light intensity of radiated celestial objects. The measured light is done by telescopes using a photometer. The photometer is composed of electronic devices such as a charged-coupled device (CCD) photometer or a photoelectric photometer that performs light conversion into electric current by the principle of the photoelectric effect. After the conversion is done, calibration is done against known intensity’s standard stars (in the background during observation). Thus, the photometer can measure the celestial object’s brightness or apparent magnitude H [ref]. The information from the measurements yields valuable information such as age, distance, temperature, and rotation period.

Early observations were done by Greek astronomers who used the Hipparchus system around 130 BC. The system divided the stars into classes called magnitudes. The brightest of stars fall in the first magnitude, followed by the second magnitude of least bright stars, and the magnitude rate increases for the fainted stars having the 6th magnitude.

In the 17th century, the discovery of fainter stars led to the invention of telescopes. This enabled further to group the fainter stars, and the magnitude group was increased to the eighth magnitude. Jumping to the 19th century saw new experiments established that the apparent equal steps of magnitude were steps of a constant ratio of light energy received during observation and that the difference in the brightness of 5 magnitudes was approximate to the ratio of a hundred. In 1856, Norman Robert proposed that this ratio govern the scale of magnitude that the brightness difference would be a ratio of 2.512 in intensity, and a 5-magnitude difference would be the ratio of 2.511885. Furthermore, steps in brightness would be denoted by decimal fractions. The zero in the scale was used to cause the minimum change of the many stars in the sixth magnitude, with many stars having magnitudes less than zero.

The invention of photography paved the way for providing a means of measuring the brightness of the stars. Photographic plates are sensitive to violet and UV radiation compared to green and yellow wavelengths that are sensitive to our eyes, thus leading to the making of two separate magnitude scales, the photographic and visual scales. Their difference is that a given star led to a colour index usage that would measure the temperature of the star’s surface. Photographic photometry relied on image comparison of visuals and recorded starlight in the plates. This led to inaccuracy due to complexity from the size and density of photographic images of the stars, and the brightness of optical images was not subject to control and proper calibration.

In the early 1940s, photometry was extended to sensitivity and wavelength range due to the application of accurate photoelectric rather than photography. The magnitude was increased to twenty-four for the faintest of stars. In photoelectric photometry, the image of a lone star s passed through a small-diaphragm in the telescope’s focal plane. Thereafter the image is passed through a selected filter and a field lens. Then the images go into a photomultiplier, a device that produces a strong electric current from a weaker light input. The output current can be measured in many ways. The ways are possible due to the extreme accuracy of the linear relationship between the incoming radiation and its current, making it possible to measure the current. The addition of photomultiplier tubes is sub-planted by the CCDs enabling the measurement of magnitudes in the visible, UV and infrared spectrum [ref].

Nowadays, photometry is more advanced and has a wide range of activities. The photometry of stars was further extended to other celestial objects like planets, comets, asteroids, galaxies, and nebulae. The measurements use apparent magnitude in the scale of magnitude per arcsecond^2 [ref]. Further knowledge of the area and light intensity across the objects can determine the surface brightness, while integrating the total light of the extended object can give the total magnitude or luminosity per unit surface area.

Photometry has numerous applications in the field of astronomy. The measurements can be merged with the inverse square law when the distance determines luminosity or the opposite. The inverse-square law is any scientific law stating that a specified physical quantity is inversely proportional to the square of the distance from the source of that physical quantity [ref]. Physical properties like temperature and material composition can be determined from either broad or narrow band spectrophotometry. One of the major applications that is part of the thesis study is detecting the light variations from asteroids and other minor planets. The measurements from the variations are then used to determine the rotation period of the asteroid. Different outputs from the variation measurement include the orbital period and radii of objects of an eclipsing binary star system and obtaining the total energy given out by a supernova [ref]

## CCD Photometry

CCD photometry uses a CCD camera that is made up of a grid of photometers that measure and record photons coming from all sources in the FOV. Since each CCD image records multiple objects at once, many forms of photometric extraction include relative, absolute, and differential. All three methods need the raw image magnitude extraction of the target objects and a known object for comparison. The observed signal of an object covers many pixels and follows the point spread function (PSF) of the system. PSF describes the response of an imaging system to a source point, also known as the system’s impulse response of a focused optical system. The broadening is due to the telescope’s optic and the astronomical seeing (the degradation of an image from turbulent flows from the Earth’s atmosphere).

When obtaining photometry from a point source, aperture photometry measures the flux by summating all the light recorded and subtracting the light due to the sky [ref]. The result is the output of the raw flux of the object. In the case of a crowded field like a cluster, de-blending techniques are used, such as PSF fitting due to the stars overlapping each other [ref].

### Calibrations

In calibration, the object’s flux is converted into instrumental magnitude (uncalibrated apparent magnitude) from counts. The calibration method depends on the photometry type used. Most observations are done using either differential or absolute photometry. The commonly used methods are absolute, relative, differential and surface photometry. Accurate photometry is rarely used as it is more difficult when the apparent brightness is fainter for an object.

### Absolute Photometry

Absolute photometry is the measurement of the apparent brightness of an object on a standard photometric system. The measurement can then be compared with other absolute photometric measurements obtained with different telescopes or instruments. Similarly, the absolute magnitudes can be compared with magnitudes from catalogues coming from other observers. Absolute photometry has the difficulty to work with high precision, unlike differential photometry.

Absolute photometry is done by correcting the differences between the effective passband through an object and the passband that defines the standard photometric system. The effective passband is the measurement of a standard star in a varying sky location, thus requiring good and stable atmospheric conditions.

### Relative Photometry

Relative photometry is the measurement of the apparent brightness of multiple objects relative to each other. It is done by comparing the instrument’s magnitude of the object to that of a known comparison object and then correcting the measurements for spatial variations in the instrument’s sensitivity and the atmosphere extinction. It also adds temporal variations when compared objects are far apart in the sky for simultaneous observations. Performing calibration of images containing both target and comparison object on proximity will result in the measurement variations decreasing to null since the photometric filters will match that of the catalogue magnitude of the comparison object. [ref]

### Differential Photometry

Differential photometry is the measurement of the difference of two observed objects, ensuring the variations of the flux of the object that are not intrinsic to it are thereby corrected. Differential photometry can be done with the highest precision, giving it an advantage over absolute photometry. The change in magnitude in equation 3.1 is important when plotting the change of magnitude of the target object, creating a light curve. This method will be used in obtaining the rotation periods of the NEAs.

This method includes airmass variations that appear in the FOV (can include clouds as well). Therefore, the comparison star should be close to the target to bear similar airmass and atmospheric conditions. It is recommended that the comparison stars have spectral types like the Sun’s one for asteroids to avoid differential extinctions. However, this effect can be lowered by averaging the signal of many stars’ comparison.

The comparison of a star relies on at least two ways of checking if the signal of the other star is stable to make it sufficient. Some stars have varying brightness and should not be used with this method. An added advantage of using many comparison stars is the reduction of noise in the measurements. This also improves the accuracy of measuring small variations.

### Surface Photometry

Spatially extended objects like galaxies are measured in the spatial distribution of brightness within the galaxy rather than the total brightness of the galaxy. A givens object surface brightness is defined as the brightness per unit solid angle as seen in a projection on the sky. This method is known as surface photometry. It is best utilised when measuring a galaxy’s surface brightness, for which the surface brightness is given as a function of the distance from the galaxy’s centre. Square Arcseconds unit is used for small solid angles. The brightness is often expressed in magnitude per square arcsecond [ref].

### Aperture photometry

Aperture photometry is the measurement of an object’s flux from an image. It involves taking the total flux in a measuring aperture and normalising it to an exposure time of a second. The background flux is approximated by using the median or average of the flux in an annulus, centred like the total flux but separated by a dead zone in the measuring aperture

## Other techniques

Besides photometry, there exists other techniques to study asteroids that provide other useful features. These techniques are not part of the scope of the thesis but present a broader picture of other ways to learn more about asteroids.

1. *Asteroid spectroscopy* aims primarily to identify features in asteroid spectra to understand their surface composition better and link them to meteorite analogues. It works on distinct parts of the electromagnetic spectrum. Many observations lie in the visible range due to the atmosphere transparency at the visible wavelength range. The visible spectrum helps to distinguish material composition, especially carbonaceous asteroids, from the silicaceous ones. Spectroscopy in the near-infrared range is more difficult due to the strong water vapour absorption. However, current observation relies on those wavelengths because of the numerous absorption bands from different mineralogical compounds found there.
2. *Asteroid polarimetry* is the study of the effects of light polarisation after it reflects over an asteroid’s surface. The partial polarisation of the unpolarised light of the Sun depends on the texture and composition of the surface, the wavelengths, and the phase angle. The phase angle of an object is the angle between the Sun, the object, and the observer. Polarimetric observations help derive the geometric albedo, size, and taxonomic classification of an asteroid.
3. *Infrared radiometry* measures optical radiation within the IR band (from 4 μm to 30 μm) in the specific windows where the atmosphere is relatively transparent. Its applications include the determination of the size, thermal inertia, surface roughness and emissivity of asteroids. The size is derived from the measurement of the disk-integrated thermal infrared flux of the asteroid, which depends on the square of its diameter. Space-based telescopes are not limited by atmosphere transparency and thermal background. Therefore, these telescopes can observe much fainter asteroids using smaller uncertainties.
4. *Radar imaging* is the analysis of the reflection of microwaves or radio waves on the asteroid surface. The time delay gives a direct measurement of the distance. Thus, the combination of both optical and radar observations allows computing fully accurate orbits. To produce the 2-D images, one must measure the distribution of echo power in time delay and Doppler frequency. Moreover, suppose images are obtained at enough viewing geometries. In that case, a 3-D shape can be derived as well as the rotation state (Ostro et al. (2002)). The echo power is proportional to the inverse fourth power of the distance; hence the technique is achievable for NEAs and the most prominent members of the MBA only. The most used radar astronomy facilities for such measurements are the Arecibo Planetary Radar and the Goldstone Solar System Radar (Ostro et al., 2002).
5. *A stellar occultation* is an event that occurs when the asteroid passes in front of a star and temporarily blocks its light as seen from Earth. The duration of this event, combined with the apparent velocity of the asteroid on the sky plane, allows deriving a physical length (called a chord) on the asteroid 2-D disk. When detecting several chords from various locations, the output becomes a 2-D profile. The profile is valid for the asteroid on the sky plane at the time of the event. Observation of stellar occultations ranges from small aperture to targets with small angular sizes such as the TNOs. The main difficulties with stellar occultation observations come from the uncertainties in the star and the asteroid positions. Many observers are thus needed to cover a large geographical area.

Moreover, those events are rare and thus hard to reproduce. The accuracy of a chord measurement depends on the accuracy of the timing of the star disappearance and reappearance. A big source of error comes from the absolute timing between chords from different observers when merging them. The stellar occultation technique is used to determine the sizes of asteroids. Still, if one obtains enough chords, it can provide details on their shapes. In rare cases, detection of atmospheres and rings is possible. It is also helpful to set the scale to a dimensionless 3-D shape.

## Photometric measurements

The common property that is crucial to obtain from an asteroid is its rotation period. Majority of asteroids in the taxonomy appear to have their rotation periods ranging between 2 to 24 hours, while the minority have lengthy rotations that span several days. Another factor that affects these varying rotation periods is the diameter. Asteroids with diameters less than 150m appear to have shorter periods around 2,4 hours. Suppose the rotation period is below 2.4 hours. In that case, the centrifugal force grows stronger for an asteroid to maintain its integrity due to the rubber pile structure. Below are discussions on the various properties of photometry.

### Rotation

Its angular moment vector , governs the fundamentals of an asteroid’s rotation, where the spin vector gives the instantaneous speed and axis rotation,also known as the angular velocity vector. The angular moment is defined in equation 3.2.

The in equation 3.1 is the f inertia tensor and is further broken down in eq. 3.3. Both the and the change over time due to collisional events and other asteroid evolution processes.

The terms and are moments of inertia along the axis x, y, and z, while the rest of the terms are the inertia products. The moment of inertia tensor explains how an asteroid’s mass is distributed around any instant spin rotation axis. Any asteroid shape has a given unique XYZ coordinate system. Every non-diagonal term from equation 3.2 becomes zero, as shown in equation 3.3. The diagonal terms now become the principal moments of inertia, and their corresponding axes are the principal axes of inertia. The principal axis governs the rotation of asteroids over a lengthy period and is further explained in section 3.3.2 [ref].

### Principal axis

The of an asteroid does vary with time because of the change in the inertia moment about the instantaneous rotation axis. It is not a constant due to changes in the inertia moment about the instantaneous spin axis. Asteroids that bear this feature are referred to as non-principal axis rotators have the and the not aligned. The unalignment leads to a complex and non-periodic rotation. The rotation state can also be referred to as an *exciting rotation*. Such a motion creates a cyclical stress/strain on the body. However, due to the nature of asteroids not truly being rigid bodies but instead composed of elastic and loosely structured materials, the interior dissipates the rotational energy. With time, the rotation attains minimal rotational energy causing the rotation to occur on the principal axis with the largest moment of inertia.

The damping timescale of a non-principal axis rotation of an asteroid is expressed as follows based on the contributions from Harris [ref]. The period is the rotation period of the asteroid and is expressed in hours. At the same time, is the mean diameter of the asteroid expressed in kilometres, and is a constant, usually approximated to the value of 17.

The value of can range from a thousand to billion years. Most asteroids have a smaller resulting in most of them rotating in their principal axis rotation states. Furthermore, this results in the possibility to measure the rotation period using light curve photometry. Light curve observations do not require much cost but demand more time for observation, for which most of the observation is done at night. These observations are done over several years to provide a clear solution of .

### Rotation measure

The of an asteroid is a calculation done by using the and for a given estimate of inertia that is based on the size, shape and density of the asteroid being measured. An accurate measure of the moment of inertia estimate uses in situ measurements. The reason the is crucial for obtaining the is because cannot be measured from any ground-based observation centre.

Ground based techniques alongside radar and optical measurements are used in the asteroid’s spin vector characterisation. Asteroids in a family that are very reflective can be measured using radar imagery. Radar imagery helps to obtain information on the shape and rotation vector of the asteroid.

### Air Mass

Observations of asteroids are affected by air mass due to the variation of the air amount in any direction when observing an asteroid or other celestial bodies. Therefore, air mass is the light’s path length that goes through the atmosphere and reaches the observer and relates to its attenuation by air. Air mass is greater when near the horizon than in the zenith position (equal to 1), increasing the measurement flux. This is because light travels on a long path for a target observed near the zenith region, resulting in dim data. Air mass can be shown in a simple equation where is the zenith distance

The cause for this increment in air mass is due to changes in humidity, air pressure, clouds, haze, and air pollution. Therefore, if light travels inside the air before detection would result in a lower flux. Air mass also causes the scattering of blue light, making the red light prominent when viewing objects.

Air mass leads to extinction which is the sequential scatter and absorption of electromagnetic radiation by gas and dust between the emitting celestial body and the observer [ref]. Observers on earth experience extinction from the interstellar medium and the Earth’s atmosphere (dust mainly). As mentioned earlier, red light is less attenuated, leading to interstellar redding [ref]. Lower altitudes areas are usually not chosen due to differential extinction across frames, making the object smaller like a mini spectrum and the position of blue stars changes to that of red ones. This effect is also due to differential fraction and higher scintillation that makes the data noisier (further discussed in section 3.3.6).

Astronomers apply the first-order extinction when there appears to be a dimming effect of the celestial’s object light. Like air mass, extinction increases as one moves closer to the horizon due to light scattering. It is a wavelength-dependent factor. The units for the first order extinction are of the degree of magnitudes per airmass. A second-order extinction focuses on the target’s colour and the airmass present at that time. Like the first order, second order extinctions add a third measurement unit called the colour index. A colour index is the difference of the magnitudes from the different filters in use at the moment of observation. [ref].

### Seeing

Seeing is the phenomenon that measures turbulence from the atmosphere due to the temperature differences in the layers of the atmosphere. Seeing is a factor that affects the accuracy of the observation. Ideally, the target object should observe it using a light source, though this is not possible for ground observation. When the layers of the atmosphere have varying temperatures, the light path would not appear parallel. The final light reaches the observer at a non-zero angle due to the difference in the refraction indexes from the layers of the atmosphere. The layers also cause the object to appear large in the image and occupy most image pixels. The SI unit for seeing is arc seconds and presents the object’s size at a Full Width at Half Maximum (FWHM) of the object’s profile, and ranges from 0.5~3.0 arc seconds [ref].

### Scintillation

Scintillation, also known as twinkling, is a phenomenon that arises when small air cells in the atmosphere cause the light of an object to arrive in several packets. It is causing the sparkling of light observed in the night sky. It is because the light is reaching the observer not in a simple packet but in several. Small apertures cause an object to twinkle, resulting in fast variations of brightness. Stars do twinkle because they are far away from the Earth, having their light sources disturbed by the atmosphere causing light diversion. Therefore, using a large aperture and longer exposure times would lower the scintillation effects [ref].

### Opposition effect

The opposition effect can help to determine the asteroid’s surface composition. As the asteroid approaches a position in its orbit, that means low phase angles (the angle Sun-asteroid-Earth), a non-linear rise of about 0.3 to 0.5 magnitudes occurs [ref]. This happens because the particles in the asteroid’s surface scatter the sunlight, and it is independent of rotational variations. [ref] Important factors are the roughness of the surface, the size, the shape, and the porosity of the particles. Thus, low albedo results in little “opposition effect.” However, the brightness of an asteroid can be different from opposition to opposition due to different obliquity. This means that the angle between the asteroid’s spin axis and the ecliptic plane is not always the same from one opposition to another. The larger the angle means more area of the asteroid’s surface is lighted, and the asteroid is brighter at that time.

## Light curves

A light curve is a light-intensity graph of a celestial object as a function of time. In comparison, light curve photometry is an asteroid’s optical observation, done over time, to find its rotational properties. Since asteroids spin around their axes, they appear to have variations of light bouncing off their surfaces during observations. These variations in light are what builds on the foundation of the measurement to generate the light curve. The light curve plot comprises a vertical axis that shows the magnitude and the use of flux in some cases. In the vertical, phase angle is used, or time to show one complete asteroid rotation. Therefore, an observer can determine the asteroid’s rotation period because of its spinning and the amplitudes present in the plot. Fig. 3-1 shows a plot of the asteroid 201 Penelope that was created from the observation obtained from the Mount John university observatory. Its rotation period was found to be 3.7474 hours.

## Chart, line chart, histogram Description automatically generated

Figure 3‑1: The Light curve of the asteroid 201 Penelope. [ref]

One light curve that covers a whole rotation can determine the asteroid’s period, but this will come at the cost of the accuracy of the period. Therefore, more light curves from multiple nights of observations can be merged into a single light curve plot increasing the amount of data of the asteroid. Eventually, it is possible to have a complete coverage of the rotation curve, especially for a slow rotating asteroid or when visibility windows of the asteroid are short. Consecutive nights of observations do have a minimum number of revolutions that can output the number of possible aliases. An alias of a period is another period that also seems to fit in the same data. Also, observations made over weeks or months contain many rotation cycles that can be used to improve the accuracy of the rotation period. Not only is the period data refined, but the data can aid in the derivation of the asteroid’s shape and its spin axis.

Light curve measurement can be done using inexpensive equipment on the ground, but they need an increased observation time. Therefore, the amateur astronomy community plays a huge role in gathering and distributing the data. Data collection from the observations is saved in databases containing the asteroid’s physical features, mostly photometric data. These databases can be accessed online. The thesis uses the Light Curve Database (LCDB), governed by the ALCDEF. The databases include;

* ALCDEF stands for Asteroid Lightcurve Data Exchange Format. The planetary science division in 2010 to function as a central repository for the submission and exchange of asteroid light curve data. The exchange format of ALCDEF comprises metadata arranged in ASCII format. The information contained in each file has the observation data of the asteroid at multiple times. Photometry data is raw with no reduced unity distances nor additions to arbitrary magnitudes. Julian Dates JD is used to show the dates of the observations. Apart from all that, the remaining data includes the asteroid’s name and number, data set contributor as anyone can contribute their data and other optional data from researchers who have studied the asteroids. Five of the asteroids selected for the thesis use data from ALCDEF [ref].
* The Asteroid Light Curve DataBase (LCDB) contains sets of files from a dBase IV database if data from either directly from observations or indirectly from calculations based on orbital characteristics, which are used to obtain the period and amplitude of an asteroid’s light curve. The information in the database comes from several sources of astronomers and numerous journals. Its main goal is to be a central repository that offers basic information of asteroid’s rotation rates and related information that can be used for research and statistical studies. The data includes estimated/measured diameters, absolute magnitudes, phase slope parameters, albedos, semi-major axis, orbital eccentricity, orbital inclination, family by name, rotation period, etc. More than 4500 asteroids are available in the database. LCDB has its online query form. The user can retrieve all columns from the LCDB Summary table and limit results to a specific group, size, and rotation period [ref].
* The Asteroid Photometric Catalogue (APC) APC contains over 8700 light curves of over 1200 asteroids in its database. It is a successor to the Standard Asteroid Photometric Catalogue (SAPC), developed by the University of Helsinki. It is an open web interface to an asteroid lightcurve database serving as a public forum for observers and researchers to exchange and share related data. APC matches modern standards and introduces new technical solutions that make the entire system easier to maintain [ref].
* The Database of Asteroid Models from Inversion Technique (DAMIT) was created to provide access to the latest physical models of the asteroids to the community. These models contain the shapes, rotation periods and spin axes of the asteroids. The models are created from photometric data using the light curve inversion technique. Refinement is necessary for some asteroids using adaptive optic images or occultation data. The astrometrica database aided in modelling a substantial of the asteroids using sparse photometric data. The database and its website are controlled by the astronomical institute of Charles University in Prague, Czech Republic [ref].

## Observations

Observation of asteroids on instruments that enable us to see and capture their data and the ideal instrument to observe and measure such data is by using a telescope. Both astronomers and space agencies using observatories use telescopes of the varying magnitude of power depending on the distance the object is being observed. The hemispherical dome of a telescope on a lonely mountaintop is one of the most familiar icons of science. Ground-based astronomical research telescopes around the world represent a capital investment of several billion dollars. The Hubble Space Telescope has a price tag of about 3 billion dollars, with an annual operating budget large enough to build a large ground-based telescope each year.

Telescopes collect more light than the unaided eye. They have increased angular resolution or ability to see fine detail than the unaided eye. Also, detectors on telescopes allow the study of wavelengths not visible to the human eye, thus, allowing a permanent record of the observation. For collecting light, the bigger the telescope, the better! More light allows us to see and study fainter objects or make more accurate measurements on bright objects. Bigger telescopes also have a better angular resolution, allowing finer detail to be seen. However, the full resolving power of research telescopes is usually not attained due to the harmful effects of the Earth’s atmosphere, which smears out the light from celestial objects.

### Image

A telescope forms an image in the focal plane. The simplest telescope is simply a convex lens. Suppose we put a small eyepiece near the focal plane and examine the image with our eye. In that case, we have a simple visual telescope of an image formed by a telescope mirror objective, as seen without the eyepiece as shown in Fig. 3-2.

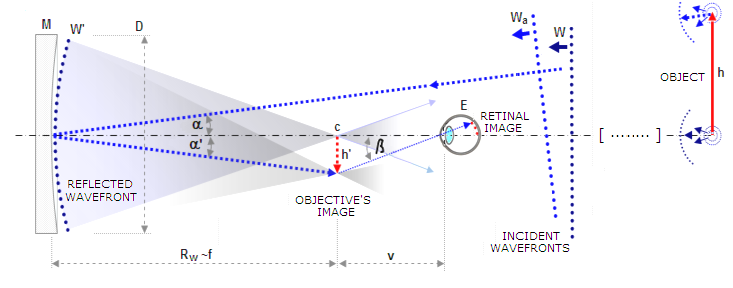


Figure 3‑2: Optical system of a Telescope. [ref]

Wavefronts arriving from a distant object of height *h* through the concave mirror *M* become flat over the aperture of a telescope. The concave mirror changes the incoming flat wavefront section *W* shape into a spherical wavefront *W`*. It does so by delaying reflection of the points in phase belonging to the wavefront’s inner area. The focal point *c* is at the centre of curvature *R* of the wavefront. Thus, with the stop at mirror surface and exit pupil plane at mirror vertex, the mirror focal length equals the radius of the wavefront in the pupil. The top point of distant object *h*, at an angle from the optical axis, is imaged into (reversed) top of the image *h`* by off-axis wavefront originating at the object’s top point. If observed directly, from the least distance of distinct vision *v*, most of the light from the object image *h`* misses the eye pupil. Also, magnification is limited to the focal length ƒ over the distinct view *v*. The apparent image angle determines objective magnification as;

### Focal length

The focal length *f* is a major characteristic property of imaging systems. Focal length is the distance from the eyepiece to the image plane. Another useful number characterising a telescope is the as shown in equation 3.10,

where is the diameter of the primary lens. The *f* sets the size of the image, while controls the amount of light in the image. Systems with a low have a large amount of light in their images, compared to the size of the image. Systems with large have low amounts of light in their images. A system’s *f* is crucial for mapping angles in the sky and linear distance in the image plane. Consider two points of light separated by an angle on the sky. The linear distance s between the points in the image is given by

provided is measured in radians and is reasonably small. Traditionally, the mapping between angle on sky and distance in the focal plane is given by the inverse plate scale, measured in units such as arcsec/mm or arcmin/mm. For systems with two mirrors, neither plane, the idea of the *f* is more complicated. This results in a system with an effective *f* much larger than the *f* of the primary mirror. The image scale and f-ratio are the effective *f*, not the primary mirror focal length.

### Telescope types

Telescopes can be divided into refracting, reflecting, or catadioptric. Refractors use a lens (transmissive optical element) as the primary light-gathering element. Reflectors use a mirror as the primary light-gathering element. Catadioptric telescopes use both transmissive element(s) and reflective element(s) as part of their primary light-gathering element. The largest refractors, built in the late 19th and early 20th centuries. Larger refractors than these have never been built due to some factors. First, since the light must pass through the lens, it must be supported only along the edge of the glass. A large lens can flex as the angle between the lens and the pull of gravity changes, distorting the figure of the lens. Refractors suffer from chromatic aberration, meaning that light of different wavelengths come to a slightly different focus. Chromatic aberration can be mitigated by using two or more elements or separate pieces of different types of glass. By proper choice of glasses with different indexes of refraction vs wavelength curves, the chromatic aberration of one element can help cancel that of another element. However, using two or more elements has disadvantages, such as increased cost and reflection light losses at each air-glass interface. Multi-element transmissive optics are used exclusively as camera lenses. Still, in present-day astronomy, refractors are rare, except for a small minority of amateur telescopes.

A Schmidt camera is often used to get nice images of large fields of degree. This uses a spherical primary. Of course, a spherical mirror suffers from spherical aberration because rays hitting the central part of the mirror comes to a different focus than rays hitting the outer parts of the mirror. In a classic Schmidt camera, a weak transmissive corrector is used, which is figured to cancel the spherical aberration of the primary. This gives good images over fields of many degrees but at the expense of a curved focal plane.

### Field of View (FOV)

The FOV is the sky area covered by an image taken with a telescope. It depends on both the focal length of the telescope and the area of the imaging detector. The angular area covered with single CCD detectors tends to be smaller with larger telescopes, as larger telescopes usually mean longer *f*.

This is related to angle the same way area is related to length. Usual units of angular area are square degrees or square arcmin. The natural unit of solid angle is the steradian, which is the angular area subtended by an angular area 1 radian by 1 radian. There are 4π steradians in a complete sphere, as seen from the sphere’s centre. The FOV of moderate to large telescopes is often much smaller than one might initially expect. One would have to take over one million images with this telescope and CCD combination to cover the entire sky.

To overcome small fields covered by CCDs at large telescopes, astronomers build cameras with multiple CCDs in the same plane. These cameras are costly and require financial and engineering resources that only the largest observatories can muster.

### Resolution

A point source has no angular extent. Although real stars have a finite angular extent, they are point sources for practical purposes with optical telescopes. So, is the image of a star a point? No. First, the atmosphere smears out the light from a point source, a particularly important and deleterious process called seeing. A telescope like the Hubble would not focus on a point source in a point image because light acts as a wave. The waves from distinct parts of the mirror interfere with each other causing an Airy disk pattern. The Airy disk has a central peak, then a series of dark and light annuli.

The angular size of the Airy disk pattern on the sky is set only by the diameter of the primary lens, not by its *f*. The linear size of the Airy disk in the image plane is set by the angular size and the image scale, set by the *f*. The angular radius of the first dark ring is given by

where λ is the wavelength of the radiation. Note that the larger the telescope primary, the smaller the angular size of the image of a point source. The angle above is traditionally called the Dawes limit or the diffraction limit. To first order, two-point sources with angular separation larger than the Dawes limit are resolvable. In contrast, two-point sources closer to the Dawes limit would be seen as one point and would not be resolvable. In practice, at least in the optical with most telescopes, angular resolving power is set by the seeing or smearing by the atmosphere, and the Dawes limit plays no role. However, this does not mean that the Dawes limit is not important. For example, the Hubble Space Telescope angular resolution is set by the Dawes limit. Of course, the Dawes limit assumes the optics are correctly figured. When Hubble was first used, the optics suffered from spherical aberration.

The smaller the angular size of a point source, the easier it is to resolve, or separate, 2-point sources close together in the sky. A 1-meter telescope observing yellow light would have a Dawes angle of about 0.14 arcsec, so the two stars 0.92 arcsec apart would be very easily resolved. Thus, in the absence of additional sources of image degradation, the 1-meter telescope could easily resolve the 0.92 arcsec stars and resolve stars about seven times closer than this, which would not be possible with the 0.15-meter telescope.

### Mountings and Locations

Most research telescopes are on general-purpose mountings that allow them to point at any spot on the sky, track the apparent motion of the stars caused by the Earth’s rotation. Some special purpose telescopes can only look at restricted parts of the sky, such as transit telescopes.

The essential general-purpose telescope mounting consists of two rotational axes at right angles to each other. For an altitude-azimuth mounting, one rotational axis points straight up, and the other axis is horizontal. We can move the telescope in altitude and in azimuth. In the equatorial mounting, one axis is tilted to be parallel to the Earth's rotational axis.

Both types of mountings have their advantages and disadvantages. An equatorial mounting allows the stars to be tracked by driving only one axis at a constant rate of once per sidereal day. In an altitude-azimuth mount, you must move both axes simultaneously to follow the paths of stars across the sky. The rates that the two axes are driven are different, and both change with position in the sky. The optical field for an equatorial mount stays at a constant angle relative to the telescope tube. In an altitude-azimuth mount, the field rotates relative to the tube.

For an altitude-azimuth mounting, this would just mean parking the telescope at the point due north, at an altitude above the horizon equal to one latitude. The stars would describe little circles around the true pole position. For an equatorial mounting, the telescope would be pointed at the pole but would rotate around its optical axis so that the polar field would be fixed relative to the tube.) To make long exposures on an altitude-azimuth telescope without trailing requires a field derotator, a gizmo that rotates the camera relative to the telescope in such a way as to cancel out the rotation induced by the mounting.

This field rotation sounds like a major annoyance. Also, altitude-azimuth mountings require a computer to calculate the drive rates for the two motors as a function of position in the sky. Therefore, altitude-azimuth mountings allow the centre of mass of the telescope to be over the centre of the azimuth bearing. For most equatorial mountings, the telescope centre of mass is not over the centre of mass the azimuth bearing, due to the tilt of the azimuth bearing. Thus, altaz mountings can be made stiffer and more compact than equatorial mountings. Most large research telescopes are now made with altitude-azimuth mountings. The cost and bother of the field derotator are trivial compared to the savings in cost due to a smaller, more compact telescope structure, which saves significant funds due to the cost of building the telescope dome or other type of enclosure.

## Preliminary orbit determination

Orbit determination of objects in our solar system is possible through celestial mechanics. Over the years, it has gained interest from scientists and researchers. The orbit computation is possible by formulating the determination of a given set of quantities observed for the body at different epochs and then the computation of the position and velocity of the body during the observation period. Orbit determination methods are orbit computation methods to obtain the classical orbital elements of the body. For an asteroid, these elements are *a, e, i, Ω, ω* and *v,* which are semi-major axis, eccentricity, inclination, right ascension of the ascending node (RAAN), the argument of perigee, and the true anomaly, respectively [ref].

The development of methods in celestial mechanics that involve orbit computation came from the discovery of Ceres by Guiseppe Piazzi (introduced in chapter 1). He made it possible by follow-up observations of Ceres for 30 days, collecting twenty observations in the process. After the observation, a new problem arose where the epoch of the observation had to be set to predict where and at what time ceres can be observed again. A year later. H. Olbers and F. Zach were able to locate ceres, with the help of Gauss, after that, creating and developing the gauss method.

Like the origins of the orbit determination techniques, a similar approach will be used in the initial orbit determination as to the first development for orbital elements of a body in motion. The process involves determining the position and velocity vector at the observation time. Since asteroid observation relies on the use of telescopes, values of RA right ascension and DEC declination will be used.

There are many orbit determination methods, including the Gauss method, Laplace method, amongst others. For the project, the thesis shall use Gibb’s and Lambart’s problem methodologies to solve for the orbital elements of the asteroids. These methods use three position vectors of the asteroids in space and the time between the observations made.

### Orbital State Vectors

The state vectors are defined as cartesian vectors of the position and velocity together with the epoch, which determines the asteroid’s orbital trajectory around the Sun [ref]. In fig. 3.2, the position vector is r while the velocity vector is v. both the position and velocity vectors are in the three directions of the Cartesians plane in the XYZ axes. A reference frame defines the state vectors. For the asteroids, the Sun is used as the centre of mass, with others as the body having more mass than the asteroid, also known as the barycentre

Diagram

Description automatically generated

Figure 3‑3: The position and velocity vectors. [ref]

The position vectors represent the position of the asteroid for a chosen reference frame. In contrast, the velocity vectors represent the asteroid’s velocity at the same reference frame. If a geocentric frame is used, the state vectors appear in equations 3.13 and 3.14, where the unit vectors , and form a right-hand triad. The velocity vector is also shown as a derivative of the position vector as (r dot).

The time these vectors were measured are sufficient to present the asteroid’s trajectory through its orbital elements. And the reverse happens as the orbital elements can give the position and velocity vectors. State vectors are important in numerical integration, accounting for significant or arbitrary perturbations like drag or gravitational effects from third bodies. As well as the central body is the Sun. that said, the orbital elements are only valid at that moment they are computed. The reason is due to the continuous perturbations that affect the orbital elements. Therefore, databases have recomputed values of these elements over time to account for the changes affecting the asteroids. It is fit to say that these orbital elements are osculating elements as they coincide with the actual orbit only at the given time.

### Gibbs Method

From observations made for asteroids, the position vectors can be obtained from a successive observation period moments *t.* since the Gibbs methods require three position vectors, three successive times need to be made, that is and . Therefore, to obtain the velocity vectors based on those three times of observation, the Gibbs method is the best solution. J.W. Gibbs was an American scholar who made great contributions to thermodynamics. He produced the preliminary orbit determination that borrows a lot of inspiration from Gauss and works with only three position vectors.

The position vectors lie on the same plane, form the basis of the conservation of angular moment. Therefore, the position vectors and lie on the same plan, and scalar factors can be used when summing up one position vector, as shown in equation 3.15. the coefficients and are output from the position vectors.

The unit vector normal to the and plane should be perpendicular to the unit vector in the direction, such that the dot product is a zero, as shown in equation 3.16.

Where the unit vectors and are shown in equations 3.17 and 3.18, respectively.

Gibbs’ method introduces some notations to help determine the velocity vectors. The notations N, D and S help to simplify the long equations that derive the velocity vector and function as intermediate vectors. These vectors help show that Gibbs’ method can only work with positional vectors alone. Further derivations of these notations can be found in the appendix.

The two notations can be combined with the angular momentum, as shown in equation 3.21. For both notations and are equal to their absolute values, except for .

The S notation, like N, is shown in equation 3.22.

Finally, the velocity vector can be obtained using equation 3.23. With that, it is now straightforward on obtaining the Keplerian elements of the asteroid. All the terms we have seen in the equations are solely dependent on the position vectors.

### Lambert’s Problem Method

Lambert’s problem method is an orbit boundary-value problem constrained to two position vectors and the elapsed time. It is an extensively used method in celestial mechanics. It has been used in many algorithms, the main one being orbit determination. The difference with Gibbs’s method is that the lamberts problem uses two positional vectors and time. This makes it useful primarily when the orbit of a celestial body is not known completely. The addition of the time can also be seen as a transfer problem where the time shows how long the asteroid takes to move from the first position vector to the second one.

The Lambert problem is a great contribution to celestial mechanics, especially the two-body orbital boundary value problem, also known as lambert’s problem or Gauss’ problem. Its objective is to determine a Keplerian orbit connecting two positions for a given time. Fig 3-4 has the central body F representing the Sun, and the origin of the reference frame, while are the positions that the asteroid occupies at the times and , where time is greater than the former.

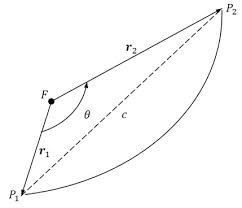


Figure 3‑4: Lambert’s problem definition. [ref]

Therefore, the flight of time of the asteroid is the time difference between the two times as shown in equation 3.24.

The angle theta indicates the direction of the asteroid’s motion. To find the velocity needed for the transfer, one needs to find the conic section in the trajectory that connects the two positions at a certain time. In the 18th century, the TBOBVP was studied intensively. Carl Gauss, born in 1777, deduced that the use of three observations times of the state vectors, instead of two, was able to determine Ceres’ orbit accurately [ref]. His work was later merged with that of Johann Lambert, who deduced the homonymous theorem:

*Given the gravitational parameter* ***µ = GM****, the time* ***∆t*** *required to accomplish a given transfer is a function of the semi-major axis* ***a*** *of the orbit, the sum of the distances from the primary at the beginning and at the end of the transfer and the length* ***c*** *of the chord that connects such positions.* - Johann Heinrich Lambert.

His theorem can be further summarised in equation 3.25 as;

Lambert’s problem is a method that will help further confirm the orbital elements coming from Gibbs’ method and compare the two for accuracy. Further breakdown of lambert’s problem method is presented in the appendix.

## Conclusion

Chapter 3 has presented the fundamental knowledge and theorems needed to create the light curves necessary for obtaining the rotational period and the Keplerian orbital elements for each asteroid.

Photometry is a vast field in astronomy, and its analysis led to applying the inverse light curve technique to obtain the light curves. Also, the databases introduce are going to be important in getting information about the asteroid for orbit

Initial orbit determination introduces two methods that will rely on the state vectors data of the asteroids to generate their preliminary orbital elements. These methods shall have their outputs compared with the ones available at the MPC.

The next chapter presents an overview and the procedure of photometry and initial orbit determination.